Complex Number Valuation of Habitats and an Information Index of the Landscape Mosaic

"It is thought, however, that the imaginary biological systems which have been treated, and the principles which have been discussed, should be of some help in interpreting real biological forms."
Alan Turing, 1952

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Abstract. Analysis of sustainability and diversity of landscapes demand methods that quantify the composition of the mosaic in different habitats. Habitats have characteristic components of value in the context of a specific landscape, such as socioeconomic and ecologic values. We suggest that vector valuation of habitats as a complex number is an interesting approach for developing tools and a conceptual framework that allows for a deep insight over the compositional problem. We define an information index for the mosaic related to its potential variability, a measure of heterogeneity, characterized by a set of values and the probabilities of their occurrence as events. Such a conceptual framework may help assessing composition scenarios in a landscape mosaic in the context of the equilibrium manifold of an idealized system.
Key words: mosaic composition, ecological economics, complex field, variability, system manifold

A Valorização de Habitats com Números Complexos e um Índice de Informação do Mosaico de Paisagem

Sumário. A análise da sustentabilidade e da diversidade de paisagens exige métodos que quantifiquem a composição do mosaico em diferentes habitats. Os habitats têm componentes características de valor no contexto de uma paisagem específica, tais como o valor socioeconómico ou o valor ecológico. Sugere-se que a valorização de um habitat como um vector, um número complexo, é uma abordagem interessante para o desenvolvimento de instrumentos conceptuais que proporcionem uma visão mais aprofundada sobre o problema da composição. Define-se um índice de informação para o mosaico relacionado com a sua variabilidade potencial, uma medida de heterogeneidade, caracterizada por um conjunto de valores e as correspondentes probabilidades de ocorrência, como acontecimentos. Este dispositivo conceptual pode ajudar a avaliar cenários de composição num mosaico de paisagem, no contexto da variedade de equilíbrios de um sistema idealizado.
Palavras-chave: composição do mosaico, economia ecológica, campo complexo, variabilidade, variedade (de equilíbrios) do sistema.

L'Évaluation de Habitats avec des Nombres Complexes et un Indice d'Information de la Mosaïque de Paysage

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Résumé. L'analyse du développement durable et de la diversité de paysages exige méthodes qui quantifient la composition de la mosaïque dans différentes habitats. Les habitats ont des composantes caractéristiques de valeur dans le contexte d'un paysage spécifique, tels comme la valeur socio-économique ou la valeur écologique. Se suggère que l'évaluation d'un habitat comme un vecteur, un nombre complexe, est un abordage intéressant pour le développement d'instruments conceptuels qui fournissent une vision plus approfondie sur le problème de la composition. On définit un indice d'information pour la mosaïque rapportée avec sa variabilité potentielle, une mesure d'hétérogénéité, caractérisée par un ensemble de valeurs et les correspondantes probabilités de présence, des événements. Ce dispositif conceptuel peut aider à évaluer des scénarios de composition dans une mosaïque de paysage, dans le contexte de la variété d'équilibres d'un système idéalisé.
Mots clés : composition de la mosaïque, économie écologique, champ complexe, variabilité, variété (d'équilibres) du système.

Introduction

In a list of major research topics in landscape ecology WU and HOBBS (2002, 2007) refer the need of developing operational definitions and measures that integrate ecological, social, cultural, economic, and aesthetic components, in the context of landscape sustainability under a hierarchical and pluralistic view.

Benchmarking the value of a specific habitat in a landscape mosaic is a challenging task. That habitat has its own relevance as a community of species, and, as a spatial interface with other, may induce synergistic effects at the landscape scale. Such habitat will have an aesthetic value and an economic and social local relevance that may be benchmarked with number values. Such multi-dimensionality asks for a vector-valuation procedure. Some authors claim that environmental resources are measured indirectly in the accounts but the underlying asset, the pristine lake or wilderness, is not valued explicitly (ATKINSON *et al*., 1997). MILLNER-GULLAND (1999) stated that the issue of the valuation of natural resources is a particularly thorny one for ecological economics, since if ecological assets are properly valued within the economy they will be conserved. Turner and CARDILLE (2007) noticed that, with few exceptions, the consideration of ecosystem function in studies has lagged behind progress in understanding the causes and consequences of spatial heterogeneity. New developments and synthesis have been made in the last years concerning indicators for biodiversity and landscape values (*e.g.* LEITÃO and AHERN, 2002). Landscape elements such as patches are habitats of different types. The number and proportions of patch types are composition attributes – aspatial - while configuration is spatial and includes the spatial arrangement of patches (LI and WU, 2007) and the number of patch types may indicate the level of resource diversity while the proportion may determine the dominance of critical resources, including fire. Patches were defined as structural components of landscape mosaic (FORMAN and GODRON, 1981) and affect processes including fire occurrence (MOREIRA *et al*., 2001). The richness of the mosaic, a composition measure, is defined as the number of different types of habitat that occur at the landscape level (TURNER, 1989; FORMAN, 1995).

Modelling the value of a habitat in the context of landscape mosaics is something that appeals for abstraction and, doing so, we proceed to the risks of reification (BOWMAN, 2007): a philosophical fallacy meaning that an abstraction is treated as a material thing. It is said that quantitative ecologists reify, because it is not possible providing a rigorous and unambiguous definition of landscape. Situation theory is a theory of information and its key insights is that much information is always available and representable only partially (PARICK and CLARK, 2007).

Methods

Complex number valuation of habitats

Let us assume we have a mosaic with *n* different habitats and that all the components of value of an habitat that are not expressed as its economic value, may be benchmarked by an ecological value, a real positive number. We denote (a,b) the ordered pair of *socioeconomic* (*a*) and *ecologic* (*b*) values of the habitat, its vector value. Complex numbers, of the form $z = a + bi$, with *a* and *b* real numbers and $i = \sqrt{-1}$, the *imaginary unity*, a name given by the French mathematician Descartes, may be represented as $z = (a,b) \bullet (1,i)$ where the dot symbol expresses the inner product of two vectors: the value vector (a,b) and the vector $(1,i)$, the basis of the complex plane. It was Wessel, the Norwegian topographer, who adopted the representation of a complex number as a vector on the cartesian plane, as far as 1797.

With this concept – the value of a habitat in a landscape is modelled by a complex number – we suggest that the ecologic value of a habitat *(b)*, although a real number, is linked to an *imaginary axis*, where the complexity and the uncertainty of the ecologic links may be expressed with more fairness. The expression $z = a + bi$ means a vectorial sum of two entities, not an algebraic one, and so is not expressed in simple unities. The set *S* of complex values of a given mosaic with *n* habitats may then be expressed as:

$$S = \{(a_1, b_1), (a_2, b_2), \cdots, (a_n, b_n)\}. \qquad (1)$$

The absolute value of the complex number $z = a + bi$ is calculated as $|z| = \sqrt{a^2 + b^2}$, the length of the hypotenuse of a triangle rectangle calculated by the Pythagorean theorem, and expresses the distance of the point (a,b) to the origin of the plane. Such a reduction from a complex number to its absolute value, a real positive number, is a strong loss of information. It is better to deal separately with the sets of economic values $Z_a = \{a_1, a_2, \cdots, a_n\}$ and ecologic values $Z_b = \{b_1, b_2, \cdots, b_n\}$ of the habitats of the mosaic, if we want to get a deeper insight on its compositional elements, measured as proportions of an actual mosaic, or the probabilities of existence as future, idealized scenarios. Functions of one complex variable will not be considered in this paper.

An information index of the mosaic

There is no such thing as an absolute measure of information. The most widely used is Shannon entropy measure (GUIASU and THEODORESCU, 1968; COVER and THOMAS, 2006), discussed in SHANNON (1948), twenty years after the preliminary work by Hartley. KLIR (2006) claims that in generalized information theory, the concept of uncertainty is conceived in the broadest possible terms. PETZ (2008) extends the concept of Hartley's information measure to quantum information. DELAHAYE (1994) reviews other information theories and measures, namely the theory of Kolmogorov based on algorithmic information and also Bennett's logic depth theory. ROSNAY (1995) considers that information is virtual time.

In general, we may say that an information measure is a tool of measurement of information based on the extent of the events that convey some kind of syntactic or semantic insight over the quantifiable object. The reasoning we adopt is related to a generalisation of Simpson's index of dominance (SIMPSON, 1949) the formula that is at the core of the relative evenness of the mosaic (*e.g.* FORMAN, 1995).

We may build a set of characteristic Bernoulli variables for the different habitats h_i of the mosaic, using a relative extension measure - the probability of the occurrence of the habitat as an event: x_i , for $i = 1, \cdots, n$. The domain is represented by the $n-1$ simplex defined as: $x_i \geq 0$ for $i = 1, \cdots, n$ and $\sum_{i=1}^{n} x_i = 1$. The Bernoulli variables for the different habitats h_i are of the form $B(h_i) = \begin{cases} 1 \leftarrow x_i \\ 0 \leftarrow 1 - x_i \end{cases}$, where the arrow means *with probability*.

With this formalism, the characteristic Bernoulli variables for habitat h_i could be expressed as $C_a(h_i) = a_i B(h_i)$ and $C_b(h_i) = b_i B(h_i)$ meaning $C_a(h_i) = \begin{bmatrix} a_i \leftarrow x_i \\ 0 \leftarrow 1 - x_i \end{bmatrix}$ and $C_b(h_i) = \begin{bmatrix} b_i \leftarrow x_i \\ 0 \leftarrow 1 - x_i \end{bmatrix}$ respectively related to the set of economic values Z_a and ecologic values Z_b. In general we may write $C_v(h_i) = \begin{bmatrix} v_i \leftarrow x_i \\ 0 \leftarrow 1 - x_i \end{bmatrix}$ where v_i means indistinctly a positive real number. A natural measure of the information for the mosaic represented by the set of characteristic values is a measure of the potential variability of that system. Since we look for a measure of variability of the system we can choose dealing with the sum of variances of the characteristic Bernoulli variables described above. Thus we can write:

$$VAR[C_v(h_i)] = v_i^2 x_i (1 - x_i) \text{ and } V = \sum_{i=1}^{n} VAR[C_v(h_i)] = \sum_{i=1}^{n} v_i^2 x_i (1 - x_i). \quad (2)$$

Results

When the probabilities of the events change the function V defined above (2), verifies the properties of a measure, and quantifies the variability of the mosaic - the more variable a system is more information it contains and we approach a first level of heterogeneity of the mosaic. The index of information V is a naturally bounded positive function: when the mosaic reduces to a single habitat, we get $V = 0$, because there is not compositional variability at the habitat level as the mosaic is reduced to the uniform matrix, a single habitat. At the other extreme, since V is a concave function defined on the $n-1$ simplex, the measure attains a maximum value a scalar V^*. We have shown, through a lagrangian multiplier method, that the maximum point coordinates are available through the equivalent formulas:

$$x_i^* \geq 0, \; x_i^* = \frac{1}{2}\left(1 + (2-k)\frac{P_{[j]}}{S}\right) \text{ with } P_{[j]} = \prod_{\substack{i=1 \\ i \neq j}}^{k} v_i^2 \text{ and } S = \sum_{j=1}^{k} P_{[j]}, \quad (3)$$

with k meaning the cardinal of a subset of the original n variables. It is easy to check that for $n=2$ there is just the optimal solution $\left(\frac{1}{2},\frac{1}{2}\right)$, constant and independent of the characteristic values v_i, and so $k=2$. For $n\geq 4$ the maximum point of function V may have null coordinate(s), although there will always be at least three positive values ($k\geq 3$), an intrinsic barrier of dimensionality; we can also prove that $x_i^*\leq\frac{1}{2}$ for $i=1,\cdots,n$ and that the maximum point coordinates do not depend on the unities of v_i. The maximum value of V may be calculated as $V^*=\sum_{i=1}^{n}v_i^2x_i^*(1-x_i^*)$ with formulas defined in (3) and an algorithmic procedure (CASQUILHO, 1999; CASQUILHO *et al.*, 2003).

The composition vector $\left(x_1,\cdots,x_n\right)$ of an actual or hypothetic mosaic with n habitats characterized by the set S defined in (1), may be compared with optimal proportions $\left(x_1^*,\cdots,x_n^*\right)$, in the sense of Z_a or Z_b, and the information value V (V_a or V_b) of that mosaic computed and benchmarked in the real interval $\left[0,V^*\right]$. The information index V has unities, related to the squared unities of the characteristic value v_i, but we can proceed through a standardized number defined as $w=\frac{V}{V^*}$, such that $w\in\left[0,1\right]$.

Discussion

When defining research priorities in landscape ecology, WU and HOBBS (2002) asked the question: “For example, can landscape patterns be optimised in terms of both the composition and configuration of patches and matrix characteristics for purposes of biodiversity conservation, ecosystem management and landscape sustainability?”

With the methodology we describe above, we hope we gave some contribute to go further on the problem of assessing quantitative insights about scenarios of composition of the mosaic in proportions of habitats.

Assuming that valuating habitat patches as a complex number is an interesting view, the complex number $z=a+bi$ is a model for the value (economic and ecologic) of the habitat in the context of a landscape mosaic. As PONNUSAMY and SILVERMAN (2006) refer, there is a complex field that contains a real field that contains a rational field. It could be argued that the natural vector valuation of habitats include three components: economic, ecologic and aesthetic values, but we belief that aesthetic value may be included both in economic and ecologic values, at least as a first approach. The b value, linked to the imaginary axis, is a nice candidate. MAOR (2007) recalls that the number 2 has a unique place in number theory, as it is the only even prime number and perhaps the ultimate mathematical constant.

With the perspective that new quantitative methods that consider the magnitude of variability in ecosystem response variables may provide new insights (TURNER and CARDILLE, 2007), we may proceed merging actual proportions (or idealized ones, as probabilities) of different habitats in an information index of the mosaic, linked to its variability, an index in the sense of PEIRCE (1909) definition: *it represents the objects independently of any resemblance to them, only by virtue of real connections with them.*

Avoiding reification implies that we don't forget that as far as the problem is discussed here, there is no explicit spatial structure of the mosaic, conceived as an abstraction and reduced to complex values of the habitats and their proportions or probabilities, a relative extension measure. May be this conceptual framework allows for procedures that contribute to an adaptive learning process in spatial optimisation as proposed by HOF and FLATHER (2007) and helps developing an integrative research perspective in multifunctional landscapes as claimed frequently (FRY, 2001; TRESS *et al*. 2005, FRY *et al*., 2007).

Since it was introduced by the term morphogenesis through early work by TURING (1952), spatial self-organization is key to understand population stability and species diversity (SOLÉ and BASCOMPTE, 2006). The index V also may be thought as a potential function governing an idealized system, as a deformation of a functional germ under the paradigm of Catastrophe Theory in the sense of DEMAZURE (1989), setting the equilibrium manifold, in a sense that O' NEILL *et al*. (1989) also exemplified as the system manifold, under the paradigm of percolation theory. The information index V does not follow the criteria of Hartley uncertainty measure, but an analogous information measure of the mosaic, based in Shannon entropy, has been presented and discussed elsewhere (CASQUILHO *et al*., 1997). We conclude this discussion quoting Hintikka (PIETARINEN, 2007): "*The traditional game-theoretic approach to semantics has two players, Myself and Nature, who assume the roles of the verifier and the falsifier of the expressions presented to them*." For a better insight over the compositional problem of the mosaic we remind that semiotics begins with the sensible world and ends with the intelligible world (MOURÃO and BABO, 2007).

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